UNDERGRADUATE FOURTH SEMESTER (HONOURS) EXAMINATIONS, 2022

Subject: Mathematics **Course ID: 42114**

Course Code: SH/MTH/404/GE-4

Course Title: Differential Equations and Vector Calculus

Time: 2 hours **Full Marks: 40**

The figures in the margin indicate full marks

Notations and symbols have their usual meaning

1. **Answer *any five* of the following questions:** **2 x 5=10**

a) Show that $\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = \vec{0}$.

b) If $y_1 = sin3x$ and $y_2 = cos3x$ then find the Wronskian of y_1 and y_2.

c) Solve $\frac{dy}{dx} = x^2y + y$.

d) If $\vec{A} = t\hat{\imath} - t^2\hat{\jmath} + t^3\hat{k}$ and $\vec{B} = \sin t\,\hat{\imath} + \cos t\,\hat{\jmath} + \cos t\,\hat{k}$, find $\frac{d}{dt}(\vec{A}.\vec{B})$.

e) Show that x=0 is a regular singular point of the differential equation

$2x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - (x+1)y = 0.$

f) If $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$ then show that the vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar.

g) If $\vec{\alpha} = 2t\hat{\imath} + t^3\hat{\jmath} + 2t^5\hat{k}$ then find the value of $\left(\frac{d\vec{\alpha}}{dt} \times \frac{d^2\vec{\alpha}}{dt^2}\right) \cdot \frac{d^3\vec{\alpha}}{dt^3}$ at t=1.

h) Is $x = e^{5t}, y = -3e^{5t}$ and $x = e^{3t}, y = -e^{3t}$ are two linearly independent solution of the system $\frac{dx}{dt} = 2x - y, \frac{dy}{dt} = 3x + 6y$?

2. **Answer *any four* of the following questions:** **5 X 4=20**

a) Apply the method of variation of parameters to solve:

$$\frac{d^2y}{dx^2} + 4y = 4\,tan\,2x.$$

b) Test the coplanarity of the vectors $7\hat{\imath} - 9\hat{\jmath} + 11\hat{k}$, $3\hat{\imath} + \hat{\jmath} - 5\hat{k}$, $5\hat{\imath} - 21\hat{\jmath} + 37\hat{k}$.

c) Solve $x^2\frac{d^2y}{dx^2} - x(x+2)\frac{dy}{dx} + (x+2)y = 0$, given $y = x$ is a solution.

d) Find the Power series solution of the differential equation $\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + 3y = 0$ about $x = 0$.

e) Solve by the method of variation of parameters: $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = \frac{e^{-x}}{x^2}$.

f) i) Prove that $\frac{d}{dt}[\vec{p}\vec{q}\vec{r}] = \left[\frac{d\vec{p}}{dt}\vec{q}\vec{r}\right] + \left[\vec{p}\frac{d\vec{q}}{dt}\vec{r}\right] + \left[\vec{p}\vec{q}\frac{d\vec{r}}{dt}\right]$ where $\vec{p}, \vec{q}, \vec{r}$ are functions of t.

ii) If $\vec{r} \times \ddot{\vec{r}} = \vec{0}$, then show that $\vec{r} \times \dot{\vec{r}} = \vec{a}$, where $\vec{r}$ is a vector function of scalar variable t and $\vec{a}$ is a constant vector.

3. Answer *any one* of the following questions: 10 x 1=10

a) i) Solve: $(3x+2)^2\frac{d^2y}{dx^2} + 5(3x+2)\frac{dy}{dx} - 3y = x^2 + x + 1$

ii) The velocity of a particle moving in space is given by $\vec{\boldsymbol{V}}(t) = -3t\hat{\boldsymbol{\imath}} + (sin^2 t)\hat{\boldsymbol{\jmath}} + (cos^2 t)\hat{\boldsymbol{k}}$. Find the particle's position as a function of t if the position at time $t = 0$ is given by $\vec{\boldsymbol{R}}(0) = \hat{\boldsymbol{\jmath}}$.

iii) Solve: $\frac{dx}{y+z} = \frac{dy}{-(x+z)} = \frac{dz}{x-z}$ 5+3+2

b) i) Solve: $\frac{dx}{dt} - \frac{dy}{dt} + 3x = \sin t$, $\frac{dx}{dt} + y = \cos t$; $given\ that\ x = 1, y = 0\ for\ t = 0.$

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ii) If $\vec{A} = 3xy\hat{\imath} - 5z\hat{\jmath} + 10x\hat{k}$, then evaluate $\int \vec{A}.d\vec{r}$ along the curve c given by $x = t^2 + 1, y = 2t^2, z = t^3\ from\ t = 1\ to\ t = 2$ 4

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